

Trigonometric Identities

Core • Sum/Difference • Double/Half Angle

ESSENTIAL IDENTITIES

Use this sheet to rewrite, simplify, and verify trigonometric expressions.

CORE IDENTITIES

Reciprocal

$$\csc \theta = 1 / \sin \theta \quad \sec \theta = 1 / \cos \theta$$

$$\cot \theta = 1 / \tan \theta$$

Quotient

$$\tan \theta = \sin \theta / \cos \theta \quad \cot \theta = \cos \theta / \sin \theta$$

Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

HALF-ANGLE IDENTITIES

$$\sin(\theta/2) = \pm \sqrt{(1 - \cos \theta)/2}$$

$$\cos(\theta/2) = \pm \sqrt{(1 + \cos \theta)/2}$$

$$\tan(\theta/2) = \pm \sqrt{(1 - \cos \theta)/(1 + \cos \theta)}$$

$$\tan(\theta/2) = \sin \theta / (1 + \cos \theta)$$

$$\tan(\theta/2) = (1 - \cos \theta) / \sin \theta$$

SIGN CHECK Choose $\pm$ from the quadrant of $\theta/2$.

SUM AND DIFFERENCE

Sine

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

Cosine

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

Tangent

$$\tan(A+B) = (\tan A + \tan B)/(1 - \tan A \tan B)$$

$$\tan(A-B) = (\tan A - \tan B)/(1 + \tan A \tan B)$$

POWER-REDUCTION IDENTITIES

$$\sin^2 \theta = [1 - \cos(2\theta)] / 2$$

$$\cos^2 \theta = [1 + \cos(2\theta)] / 2$$

$$\tan^2 \theta = [1 - \cos(2\theta)] / [1 + \cos(2\theta)]$$

DOUBLE-ANGLE IDENTITIES

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos(2\theta) = 2\cos^2 \theta - 1$$

$$\cos(2\theta) = 1 - 2\sin^2 \theta$$

$$\tan(2\theta) = 2\tan \theta / (1 - \tan^2 \theta)$$

PATTERN REMINDERS

Sum/Difference: sine keeps the sign; cosine switches it.

Double Angle: comes from setting $A = B = \theta$.

Half Angle: comes from rearranging cosine double angle.

Power Reduction: reduces squared trig functions.

IDENTITY STRATEGY

1. Identify the trig functions and powers.
2. Look for a familiar identity or rewrite in sine/cosine.
3. Simplify one side at a time when verifying.
4. Check signs, domains, and quadrant information.