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QUICK REFERENCE**First Derivative**

$f'(x) > 0 \Rightarrow \text{increasing}$

$f'(x) < 0 \Rightarrow \text{decreasing}$

Second Derivative

$f''(x) > 0 \Rightarrow \text{concave up}$

$f''(x) < 0 \Rightarrow \text{concave down}$

Motion

$v(t) = s'(t)$

$a(t) = s''(t)$

LEVEL 1 - CRITICAL NUMBERS AND FUNCTION BEHAVIOR**1 CRITICAL NUMBERS**

Find the critical numbers of

$$f(x) = x^3 - 3x^2.$$

2 INCREASING / DECREASING

Suppose

$$f'(x) = 3x(x - 2).$$

On which intervals is f increasing?Choices: $(-\infty, 0)$ and $(2, \infty)$ $(0, 2)$ $(-\infty, 2)$ **3 FIRST DERIVATIVE TEST**At a critical number $x = c$, suppose $f'(x)$ changes from positive to negative.
What occurs at $x = c$?

Choices: Local minimum Local maximum Inflection point

LEVEL 2 - EXTREMA, CONCAVITY, AND CURVE ANALYSIS**4 ABSOLUTE EXTREMA**

Find the absolute maximum and absolute minimum of

$$f(x) = x^3 - 3x^2$$

on

$$[-1, 3].$$

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5 CONCAVITY

Determine where

$$f(x) = x^3 - 3x^2$$

is concave up and concave down.

6 INFLECTION POINT

Find the inflection point of

$$f(x) = x^3 - 3x^2.$$

7 SECOND DERIVATIVE TEST

Suppose

$$f'(4) = 0 \quad \text{and} \quad f''(4) = 6.$$

What can you conclude?

Choices: Local maximum at $x = 4$ Local minimum at $x = 4$ No conclusion is possible**LEVEL 3 - OPTIMIZATION****8 OPTIMIZATION SETUP**

A rectangle has perimeter 40 units.

If one side is x , write the area A as a function of x .**9 MAXIMUM AREA**

Using

$$A(x) = 20x - x^2,$$

find the dimensions of the rectangle with maximum area.

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10 OPTIMIZATION

A farmer has 200 meters of fencing to enclose a rectangular field against a straight river.
No fence is needed along the river.
Find the dimensions that maximize the area.

LEVEL 4 - MOTION AND RELATED RATES**11 MOTION**

An object's position is

$$s(t) = t^3 - 6t^2 + 9t.$$

Find its velocity and acceleration.

12 AT REST

For

$$s(t) = t^3 - 6t^2 + 9t,$$

find the times when the object is at rest.

13 RELATED RATES

The radius of a circle is increasing at

$$\frac{dr}{dt} = 3 \text{ cm/s.}$$

How quickly is the area increasing when $r = 5$ cm?

14 RELATED RATES

The side length s of a square is increasing at

$$\frac{ds}{dt} = 2 \text{ cm/s.}$$

How quickly is the area increasing when $s = 6$ cm?

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LEVEL 5 - LINEAR APPROXIMATION AND MIXED REVIEW

15 LINEAR APPROXIMATION

Use linear approximation at $x = 4$ to estimate

$$\sqrt{4.1}.$$

16 MIXED ANALYSIS

Suppose a function satisfies

$$f'(x) < 0 \quad \text{and} \quad f''(x) > 0$$

on an interval.

Describe the behavior of f .

Choices: Increasing and concave up Decreasing and concave up Decreasing and concave down

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Use the solutions to check your method and reasoning.

1 CRITICAL NUMBERS

Differentiate:

$$f'(x) = 3x^2 - 6x = 3x(x - 2).$$

Set the derivative equal to zero:

$$3x(x - 2) = 0.$$

Answer: $x = 0, 2$ **2 INCREASING / DECREASING**

The critical numbers divide the real line into

$$(-\infty, 0), \quad (0, 2), \quad (2, \infty).$$

The sign pattern of f' is

$$+ \quad - \quad +.$$

Therefore f is increasing where $f'(x) > 0$. **Answer:** $(-\infty, 0) \cup (2, \infty)$ **3 FIRST DERIVATIVE TEST**A positive derivative means f is increasing, while a negative derivative means f is decreasing.

The change

$$+ \longrightarrow -$$

means the graph changes from increasing to decreasing. **Answer:** Local maximum at $x = c$.**4 ABSOLUTE EXTREMA**Check the endpoints and the critical numbers $x = 0, 2$:

$$f(-1) = -4, \quad f(0) = 0, \quad f(2) = -4, \quad f(3) = 0.$$

Answer: Absolute maximum 0 at $x = 0, 3$; absolute minimum -4 at $x = -1, 2$.**5 CONCAVITY**

Differentiate twice:

$$f''(x) = 6x - 6.$$

Set $f''(x) = 0$:

$$6x - 6 = 0 \Rightarrow x = 1.$$

For $x < 1$, $f''(x) < 0$. For $x > 1$, $f''(x) > 0$. **Answer:** Concave down on $(-\infty, 1)$; concave up on $(1, \infty)$.

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6 INFLECTION POINT

The second derivative changes sign at $x = 1$, so the concavity changes there.

Evaluate the original function:

$$f(1) = 1 - 3 = -2.$$

Answer: $(1, -2)$

7 SECOND DERIVATIVE TEST

Because $f'(4) = 0$, $x = 4$ is a critical number.

Since

$$f''(4) = 6 > 0,$$

the graph is concave up there. **Answer:** Local minimum at $x = 4$.

8 OPTIMIZATION SETUP

Let the other side be y . Then

$$2x + 2y = 40 \Rightarrow y = 20 - x.$$

Area is

$$A = xy.$$

Therefore

$$A(x) = x(20 - x) = 20x - x^2.$$

Answer: $A(x) = 20x - x^2$

9 MAXIMUM AREA

Differentiate:

$$A'(x) = 20 - 2x.$$

Set equal to zero:

$$20 - 2x = 0 \Rightarrow x = 10.$$

Then

$$y = 20 - x = 10.$$

Answer: 10 units by 10 units.

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10 OPTIMIZATION

Let x be each side perpendicular to the river and y the side parallel to it:

$$2x + y = 200 \Rightarrow y = 200 - 2x.$$

Then

$$A(x) = x(200 - 2x) = 200x - 2x^2.$$

Differentiate:

$$A'(x) = 200 - 4x.$$

Set $A'(x) = 0$:

$$200 - 4x = 0 \Rightarrow x = 50.$$

Then

$$y = 200 - 2(50) = 100.$$

Answer: 50 m by 100 m.

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Use the solutions to check your method and reasoning.

11 MOTION

Velocity is the derivative of position:

$$v(t) = s'(t) = 3t^2 - 12t + 9.$$

Acceleration is the derivative of velocity:

$$a(t) = v'(t) = 6t - 12.$$

Answer: $v(t) = 3t^2 - 12t + 9$, $a(t) = 6t - 12$ **12** AT RESTAn object is at rest when $v(t) = 0$:

$$3t^2 - 12t + 9 = 0.$$

Factor:

$$3(t - 1)(t - 3) = 0.$$

Answer: $t = 1, 3$ **13** RELATED RATES

Start with

$$A = \pi r^2.$$

Differentiate with respect to time:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

Substitute $r = 5$ and $\frac{dr}{dt} = 3$:

$$\frac{dA}{dt} = 2\pi(5)(3) = 30\pi.$$

Answer: $30\pi \text{ cm}^2/\text{s}$ **14** RELATED RATES

Start with

$$A = s^2.$$

Differentiate:

$$\frac{dA}{dt} = 2s \frac{ds}{dt}.$$

Substitute $s = 6$ and $\frac{ds}{dt} = 2$:

$$\frac{dA}{dt} = 2(6)(2) = 24.$$

Answer: $24 \text{ cm}^2/\text{s}$

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15 LINEAR APPROXIMATION

Let

$$f(x) = \sqrt{x}.$$

At $a = 4$,

$$f(4) = 2 \quad \text{and} \quad f'(4) = \frac{1}{4}.$$

The linearization is

$$L(x) = 2 + \frac{1}{4}(x - 4).$$

Then

$$L(4.1) = 2 + \frac{1}{4}(0.1) = 2.025.$$

Answer: $\sqrt{4.1} \approx 2.025$ **16** MIXED ANALYSIS

Since

$$f'(x) < 0,$$

the function is decreasing.

Since

$$f''(x) > 0,$$

the graph is concave up. **Answer: Decreasing and concave up.**