

Name: _____

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QUICK REFERENCE**Area Between Curves**

$$A = \int (\text{top} - \text{bottom}) dx$$

Disk Method

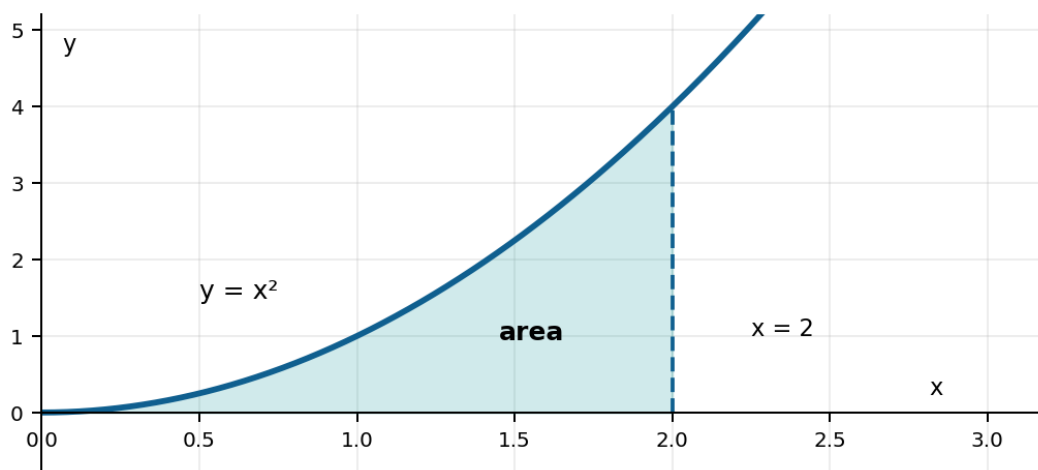
$$V = \pi \int R^2 dx$$

Washer Method

$$V = \pi \int (R^2 - r^2) dx$$

Shell Method

$$V = 2\pi \int rh dx$$

LEVEL 1 · AREA AND SIGNED AREA**1 AREA UNDER A CURVE**Find the area under $y = x^2$ from $x = 0$ to $x = 2$.**2 SIGNED AREA**Suppose $\int_0^2 f(x) dx = 5$ and $\int_2^4 f(x) dx = -3$. Find $\int_0^4 f(x) dx$. Choices: 8 2 -2**3 TOTAL AREA**The graph of f is above the x -axis from $x = 0$ to $x = 2$ with area 6, and below the x -axis from $x = 2$ to $x = 5$ with geometric area 4. What is the total geometric area? Choices: 2 10 -10

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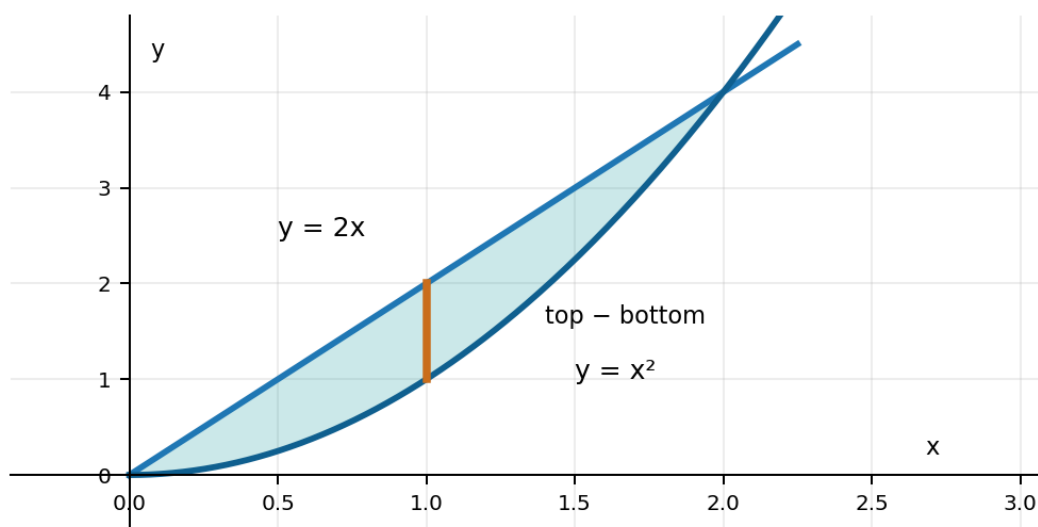
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LEVEL 2 · AREA BETWEEN CURVES

4 INTERSECTIONS

Find the intersection points of $y = 2x$ and $y = x^2$.

5 AREA BETWEEN CURVES

Find the area enclosed by $y = 2x$ and $y = x^2$.

6 HORIZONTAL SLICES

When integrating with respect to y , which subtraction order is used for area? Choices: Top minus bottom Right minus left Left minus right

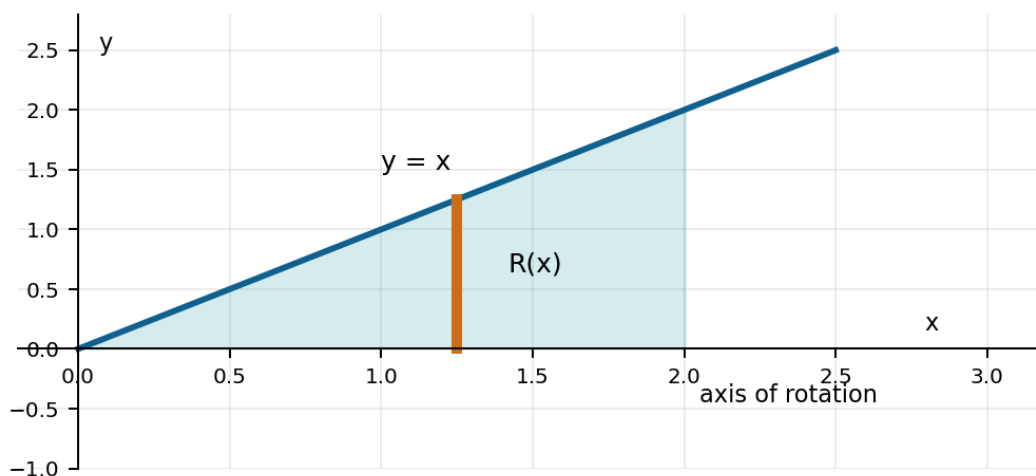
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LEVEL 3 · VOLUMES OF REVOLUTION

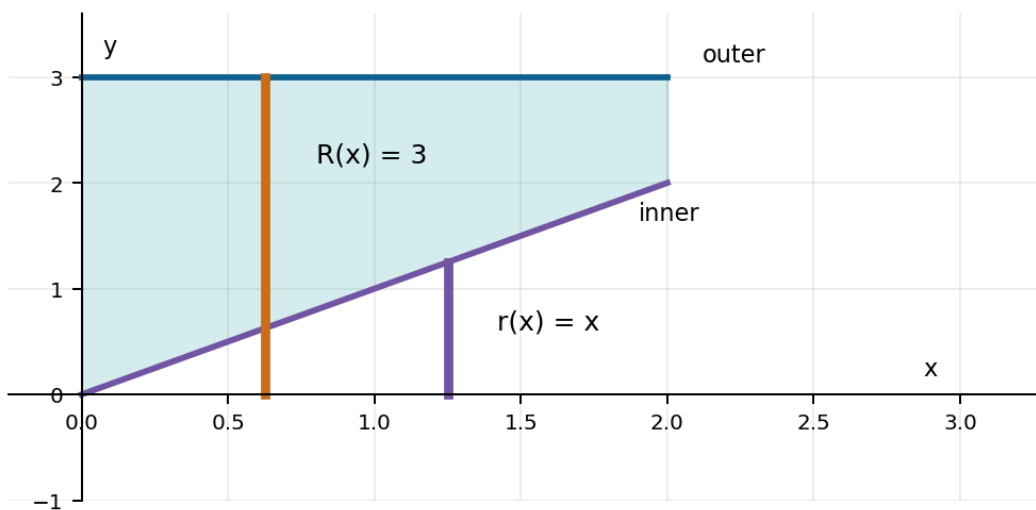
7 DISK METHOD

The region under $y = x$ from $x = 0$ to $x = 2$ is rotated about the x -axis. Find the volume.



8 WASHER METHOD

A region has outer radius $R(x) = 3$ and inner radius $r(x) = x$ for $0 \leq x \leq 2$. Write and evaluate the washer-method volume.

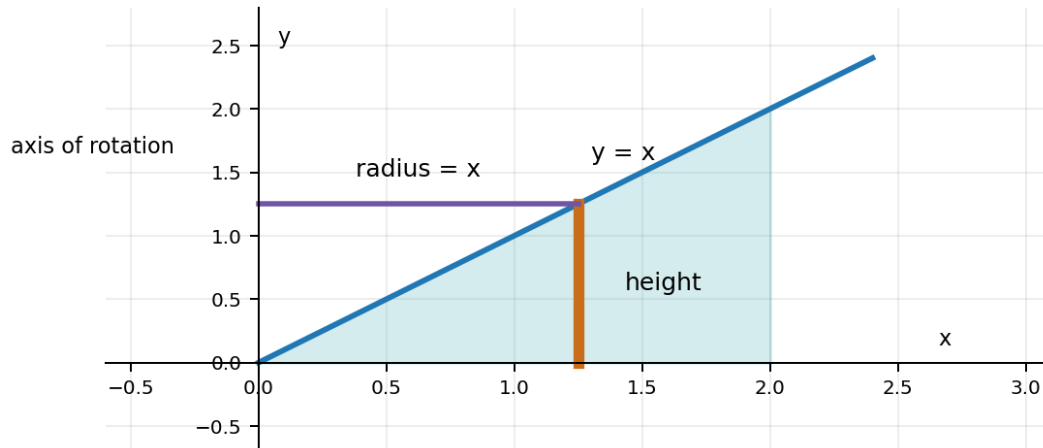


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LEVEL 3 · VOLUMES OF REVOLUTION**9 SHELL METHOD**

The region under $y = x$ from $x = 0$ to $x = 2$ is rotated about the y -axis. Use cylindrical shells to find the volume.

**LEVEL 4 · AVERAGE VALUE AND ACCUMULATION****10 AVERAGE VALUE**

Find the average value of $f(x) = x^2$ on $[0, 2]$.

11 NET CHANGE

Water enters a tank at the rate $r(t) = 4t + 2$ liters per minute. How much water enters from $t = 0$ to $t = 3$?

12 ACCUMULATION FUNCTION

Let $A(x) = \int_1^x (t^2 + 3) dt$. Find $A'(x)$.

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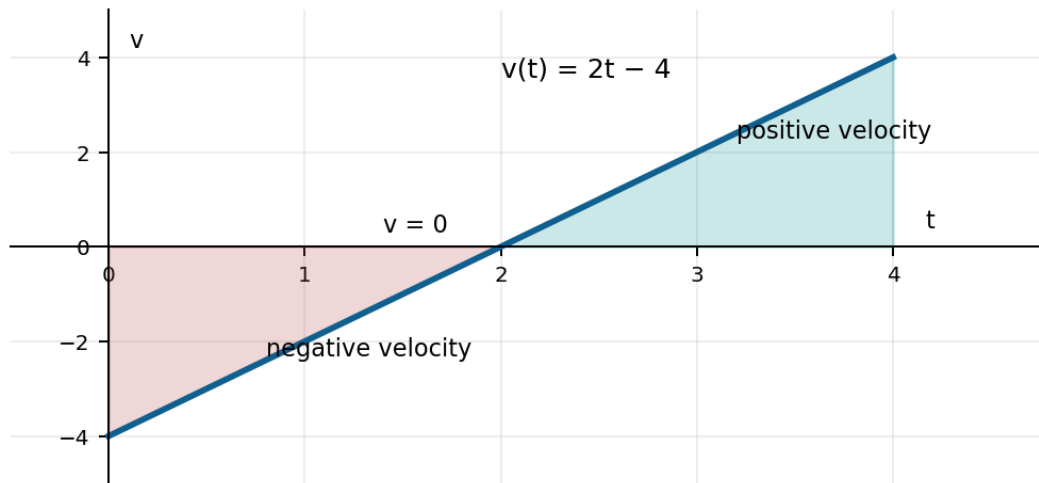
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LEVEL 5 · MOTION AND MIXED APPLICATIONS**13 DISPLACEMENT**

An object has velocity $v(t) = 2t - 4$ for $0 \leq t \leq 4$. Find its displacement.

14 TOTAL DISTANCE

For the same velocity $v(t) = 2t - 4$, find the total distance traveled from $t = 0$ to $t = 4$.

**15 METHOD SELECTION**

A region is rotated around the y -axis. Using vertical slices, which volume method naturally produces cylindrical shells?

Choices: Disk method Washer method Shell method

16 MIXED REVIEW

Which expression gives the average value of a continuous function f on $[a, b]$? Choices: $\int_a^b f(x) dx$ $\frac{1}{b-a} \int_a^b f(x) dx$

$$\frac{f(a)+f(b)}{2}$$

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Use the solutions to check your setup and reasoning.

1 AREA UNDER A CURVESince $x^2 \geq 0$ on $[0, 2]$, $A = \int_0^2 x^2 dx = [x^3/3]_0^2 = 8/3$. **Answer:** $\frac{8}{3}$ **2 SIGNED AREA**Add adjacent definite integrals: $\int_0^4 f = 5 + (-3) = 2$. **Answer:** 2**3 TOTAL AREA**Total geometric area counts both regions positively: $6 + 4 = 10$. **Answer:** 10**4 INTERSECTIONS**Set $2x = x^2$: $x(x - 2) = 0$, so $x = 0, 2$. **Answer:** $x = 0, 2$ **5 AREA BETWEEN CURVES**On $[0, 2]$, $2x$ is above x^2 :

$$A = \int_0^2 (2x - x^2) dx = [x^2 - x^3/3]_0^2 = 4 - 8/3 = 4/3.$$

Answer: $\frac{4}{3}$ **6 HORIZONTAL SLICES**Horizontal slices run from the left boundary to the right boundary. **Answer:** Right minus left**7 DISK METHOD**The radius is $R(x) = x$:

$$V = \pi \int_0^2 x^2 dx = \pi [x^3/3]_0^2 = \frac{8\pi}{3}.$$

Answer: $\frac{8\pi}{3}$ **8 WASHER METHOD**

$$V = \pi \int_0^2 (3^2 - x^2) dx = \pi [9x - x^3/3]_0^2 = \frac{46\pi}{3}.$$

Answer: $\frac{46\pi}{3}$

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Use the solutions to check your setup and reasoning.

9 SHELL METHODRadius = x , height = x :

$$V = 2\pi \int_0^2 x(x) dx = 2\pi [x^3/3]_0^2 = \frac{16\pi}{3}.$$

Answer: $\frac{16\pi}{3}$ **10 AVERAGE VALUE**

$$f_{\text{avg}} = \frac{1}{2-0} \int_0^2 x^2 dx = \frac{1}{2} \cdot \frac{8}{3} = \frac{4}{3}.$$

Answer: $\frac{4}{3}$ **11 NET CHANGE**

$$\int_0^3 (4t + 2) dt = [2t^2 + 2t]_0^3 = 18 + 6 = 24.$$

Answer: 24 liters**12 ACCUMULATION FUNCTION**By the Fundamental Theorem of Calculus, $A'(x) = x^2 + 3$. **Answer:** $A'(x) = x^2 + 3$ **13 DISPLACEMENT**

$$\int_0^4 (2t - 4) dt = [t^2 - 4t]_0^4 = 0.$$

Answer: 0**14 TOTAL DISTANCE**Velocity changes sign at $t = 2$. Thus

$$-\int_0^2 (2t - 4) dt + \int_2^4 (2t - 4) dt = 4 + 4 = 8.$$

Answer: 8**15 METHOD SELECTION**Vertical slices rotated about the y -axis form cylindrical shells. **Answer:** Shell method

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16 MIXED REVIEW

Average value equals accumulated value divided by interval length. **Answer:** $\frac{1}{b-a} \int_a^b f(x) dx$