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QUICK REFERENCE

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$
$$n \neq -1$$

Reciprocal Rule

$$\int \frac{1}{x} dx = \ln |x| + C$$

Fundamental Theorem

$$\int_a^b f(x) dx = F(b) - F(a)$$

 u -Substitution

$$u = g(x)$$
$$du = g'(x) dx$$

LEVEL 1 · ANTIDERIVATIVES AND BASIC RULES

1 ANTIDERIVATIVE

Find the indefinite integral:

$$\int 6x dx$$

2 CONSTANT RULE

Evaluate:

$$\int 7 dx$$

3 POWER RULE

Evaluate:

$$\int x^5 dx$$

LEVEL 2 · POWERS, POLYNOMIALS, AND SPECIAL INTEGRALS

4 POLYNOMIAL

Evaluate:

$$\int (4x^3 - 6x^2 + 5x - 8) dx$$

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5 NEGATIVE EXPONENT

Evaluate:

$$\int x^{-3} dx$$

LEVEL 2 · POWERS, POLYNOMIALS, AND SPECIAL INTEGRALS**6** RADICAL

Evaluate:

$$\int \sqrt{x} dx$$

7 RECIPROCAL RULE

Evaluate:

$$\int \frac{1}{x} dx$$

LEVEL 3 · DEFINITE INTEGRALS AND THE FUNDAMENTAL THEOREM**8** DEFINITE INTEGRAL

Evaluate:

$$\int_1^3 2x dx$$

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9 **FUNDAMENTAL THEOREM**

Differentiate:

$$G(x) = \int_2^x (t^3 + 1) dt$$

LEVEL 3 · DEFINITE INTEGRALS AND THE FUNDAMENTAL THEOREM**10** **FTC + CHAIN RULE**

Differentiate:

$$G(x) = \int_0^{x^2} \cos t dt$$

LEVEL 4 · TRIGONOMETRIC INTEGRALS AND u -SUBSTITUTION**11** **TRIG INTEGRAL**

Evaluate:

$$\int (3 \cos x - 2 \sin x) dx$$

12 **u -SUBSTITUTION**

Evaluate:

$$\int 2x(x^2 + 1)^4 dx$$

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13 CONSTANT FACTOR

Evaluate:

$$\int x(x^2 + 4)^3 dx$$

LEVEL 4 · TRIGONOMETRIC INTEGRALS AND u -SUBSTITUTION**14** DEFINITE u -SUBSTITUTION

Evaluate:

$$\int_0^1 2x(x^2 + 1)^2 dx$$

LEVEL 5 · INTEGRAL PROPERTIES AND MIXED REVIEW**15** INTEGRAL PROPERTIES

Suppose

$$\int_1^4 f(x) dx = 7.$$

Find

$$\int_4^1 f(x) dx.$$

Choices: 7 -7 0

16 SYMMETRYSuppose f is an odd function.

Evaluate

$$\int_{-5}^5 f(x) dx.$$

Choices: 0 5 Cannot be determined

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Use the solutions to check your method and reasoning.

1 ANTIDERIVATIVE

Use the power rule:

$$\int 6x \, dx = 6 \left(\frac{x^2}{2} \right) + C = 3x^2 + C.$$

Answer: $3x^2 + C$ **2 CONSTANT RULE**The antiderivative of a constant c is $cx + C$:

$$\int 7 \, dx = 7x + C.$$

Answer: $7x + C$ **3 POWER RULE**

Add one to the exponent and divide by the new exponent:

$$\int x^5 \, dx = \frac{x^6}{6} + C.$$

Answer: $\frac{x^6}{6} + C$ **4 POLYNOMIAL**

Integrate term by term:

$$\int (4x^3 - 6x^2 + 5x - 8) \, dx = x^4 - 2x^3 + \frac{5}{2}x^2 - 8x + C.$$

Answer: $x^4 - 2x^3 + \frac{5}{2}x^2 - 8x + C$ **5 NEGATIVE EXPONENT**

Apply the power rule:

$$\int x^{-3} \, dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C.$$

Answer: $-\frac{1}{2x^2} + C$ **6 RADICAL**Rewrite $\sqrt{x} = x^{1/2}$:

$$\int x^{1/2} \, dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C.$$

Answer: $\frac{2}{3}x^{3/2} + C$

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7 RECIPROCAL RULEThe power rule does not apply when $n = -1$:

$$\int \frac{1}{x} dx = \ln|x| + C.$$

Answer: $\ln|x| + C$ **8 DEFINITE INTEGRAL**An antiderivative of $2x$ is x^2 :

$$\left[x^2\right]_1^3 = 3^2 - 1^2 = 8.$$

Answer: 8**9 FUNDAMENTAL THEOREM**

By the Fundamental Theorem of Calculus,

$$\frac{d}{dx} \left[\int_a^x f(t) dt \right] = f(x).$$

Therefore

$$G'(x) = x^3 + 1.$$

Answer: $G'(x) = x^3 + 1$ **10 FTC + CHAIN RULE**

Evaluate the integrand at the upper bound and multiply by the derivative of that bound:

$$G'(x) = \cos(x^2)(2x) = 2x \cos(x^2).$$

Answer: $G'(x) = 2x \cos(x^2)$ **11 TRIG INTEGRAL**

Use

$$\int \cos x dx = \sin x, \quad \int \sin x dx = -\cos x.$$

Then

$$\int (3 \cos x - 2 \sin x) dx = 3 \sin x + 2 \cos x + C.$$

Answer: $3 \sin x + 2 \cos x + C$

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12 *u*-SUBSTITUTION

Let

$$u = x^2 + 1, \quad du = 2x \, dx.$$

Then

$$\int 2x(x^2 + 1)^4 \, dx = \int u^4 \, du = \frac{u^5}{5} + C.$$

Substitute back:

$$\frac{(x^2 + 1)^5}{5} + C.$$

Answer: $\frac{(x^2+1)^5}{5} + C$

13 CONSTANT FACTOR

Let

$$u = x^2 + 4, \quad du = 2x \, dx, \quad x \, dx = \frac{1}{2} du.$$

Then

$$\frac{1}{2} \int u^3 \, du = \frac{u^4}{8} + C.$$

Answer: $\frac{(x^2+4)^4}{8} + C$

14 DEFINITE *u*-SUBSTITUTION

Let

$$u = x^2 + 1, \quad du = 2x \, dx.$$

Change the bounds:

$$x = 0 \Rightarrow u = 1, \quad x = 1 \Rightarrow u = 2.$$

Then

$$\int_1^2 u^2 \, du = \left[\frac{u^3}{3} \right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}.$$

Answer: $\frac{7}{3}$

15 INTEGRAL PROPERTIES

Reversing the bounds changes the sign:

$$\int_4^1 f(x) \, dx = - \int_1^4 f(x) \, dx = -7.$$

Answer: -7

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16 SYMMETRY

For an odd function,

$$f(-x) = -f(x).$$

Over a symmetric interval,

$$\int_{-a}^a f(x) dx = 0.$$

Therefore

$$\int_{-5}^5 f(x) dx = 0.$$

Answer: 0