

Name: _____

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QUICK REFERENCE**Direct substitution**Try $x = a$ first**0/0**

Indeterminate: simplify

Two-sided limit

Left-hand = right-hand

LEVEL 1 • UNDERSTANDING LIMITS**1 READ LIMIT NOTATION****What does the following limit statement mean?**

$$\lim_{x \rightarrow 3} f(x) = 7$$

Choices: $f(3)=7$ $f(x)$ approaches 7 x approaches 7**2 FUNCTION VALUE VS LIMIT****Suppose $f(2)=1$, but the following limit is true. Which statement is correct?**

$$\lim_{x \rightarrow 2} f(x) = 5$$

Choices: The limit is 1 The function approaches 5 The limit DNE

3 ESTIMATE NUMERICALLY**Use $f(1.9)=3.9$, $f(1.99)=3.99$, $f(2.01)=4.01$, and $f(2.1)=4.1$ to estimate:**

$$\lim_{x \rightarrow 2} f(x)$$

LEVEL 2 • DIRECT SUBSTITUTION**4 DIRECT SUBSTITUTION****Evaluate:**

$$\lim_{x \rightarrow 3} (x^2 + 2x - 1)$$

5 RATIONAL FUNCTION**Evaluate:**

$$\lim_{x \rightarrow 2} \frac{x+4}{x+1}$$

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LEVEL 2 · DIRECT SUBSTITUTION

6 RECOGNIZE 0/0

What happens when direct substitution is used on:

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$$

Choices: 4 0/0 ∞

LEVEL 3 · ALGEBRAIC LIMITS

7 FACTORING

Evaluate:

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

8 FACTOR A QUADRATIC

Evaluate:

$$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$$

9 RATIONALIZE

Evaluate:

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$$

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LEVEL 4 · ONE-SIDED AND INFINITE LIMITS

10 ONE-SIDED LIMITS

Suppose the one-sided limits below are known. Find the two-sided limit.

$$\lim_{x \rightarrow 2^-} f(x) = 4, \quad \lim_{x \rightarrow 2^+} f(x) = 4$$

11 DNE

Suppose the one-sided limits below are known. Find the two-sided limit.

$$\lim_{x \rightarrow 5^-} f(x) = 2, \quad \lim_{x \rightarrow 5^+} f(x) = 8$$

12 INFINITE LIMIT

Evaluate:

$$\lim_{x \rightarrow 0^+} \frac{1}{x}$$

13 VERTICAL ASYMPTOTE

For the function below, what vertical line is a vertical asymptote?

$$f(x) = \frac{1}{x-3}$$

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LEVEL 5 · MIXED LIMITS AND CONTINUITY

14 CHOOSE A STRATEGY

Evaluate:

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$$

15 CONTINUITY TEST

Suppose $f(4)=7$ and the following limit is true. Is f continuous at $x=4$? Explain.

$$\lim_{x \rightarrow 4} f(x) = 7$$

16 DISCONTINUITY

Suppose $f(2)=1$ and the following limit is true. Is f continuous at $x=2$? Explain.

$$\lim_{x \rightarrow 2} f(x) = 6$$

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Use the solutions to check your method and reasoning.

1 READ LIMIT NOTATION

A limit describes nearby behavior; it does not require $f(3)=7$.

Answer: $f(x)$ approaches 7 as x approaches 3.

2 FUNCTION VALUE VS LIMIT

$f(2)=1$ is the value at $x=2$. Nearby values approach 5.

Answer: The function approaches 5.

3 ESTIMATE NUMERICALLY

The listed outputs approach 4 from both sides of $x=2$.

Answer: 4

4 DIRECT SUBSTITUTION

Substitute $x=3$: $3^2+2(3)-1=14$.

Answer: 14

5 RATIONAL FUNCTION

Substitute $x=2$: $(2+4)/(2+1)=2$.

Answer: 2

6 RECOGNIZE 0/0

Substitute $x=4$: $(16-16)/(4-4)=0/0$. This is indeterminate.

Answer: 0/0

7 FACTORING

Factor $x^2-9=(x-3)(x+3)$, cancel $x-3$, then evaluate $x+3$ at 3.

Answer: 6

8 FACTOR A QUADRATIC

Factor $x^2-x-2=(x-2)(x+1)$, cancel $x-2$, then evaluate $x+1$ at 2.

Answer: 3

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Use the solutions to check your method and reasoning.

9 RATIONALIZE

Multiply by the conjugate. The expression becomes $1/(\sqrt{x}+2)$, then substitute $x=4$.

Answer: $1/4$ **10 ONE-SIDED LIMITS**

Both one-sided limits equal 4, so the two-sided limit exists.

Answer: 4**11 DNE**

The one-sided limits disagree: $2 \neq 8$.

Answer: DNE**12 INFINITE LIMIT**

As x approaches 0 from the right, $1/x$ increases without bound.

Answer: $+\infty$ **13 VERTICAL ASYMPTOTE**

Set $x-3=0$.

Answer: $x=3$ **14 CHOOSE A STRATEGY**

Factor $x^2-25=(x-5)(x+5)$, cancel, then evaluate $x+5$ at 5.

Answer: 10**15 CONTINUITY TEST**

The function value exists, the limit exists, and both equal 7.

Answer: Yes.**16 DISCONTINUITY**

The limit is 6 but $f(2)=1$, so the continuity condition fails.

Answer: No.