

Name: _____

Date: _____

Page 1

QUICK REFERENCE

Geometric Series

$$S = \frac{a}{1-r}, |r| < 1$$

 p -Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

$p > 1 \Rightarrow \text{converges}$

Ratio Test

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$L < 1$ converges, $L > 1$ diverges

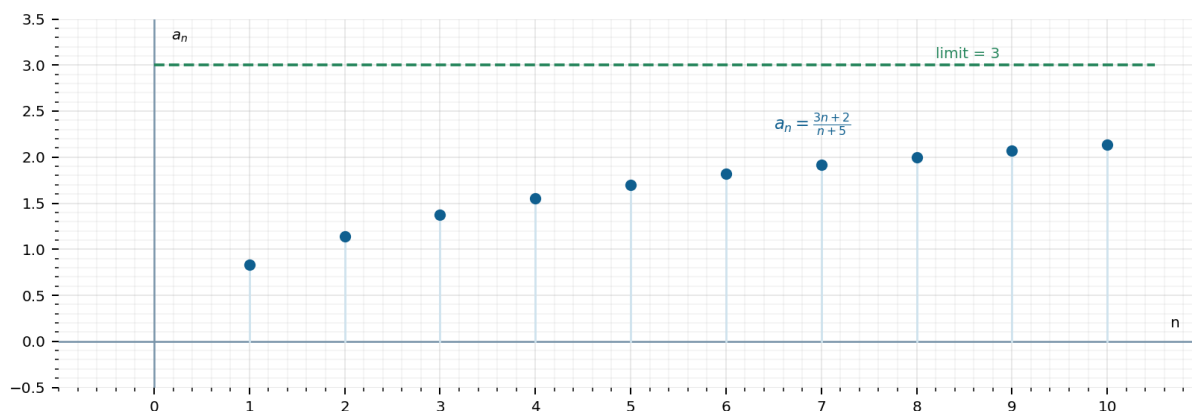
LEVEL 1 · SEQUENCES AND GEOMETRIC SERIES

1 SEQUENCE TERMS

Suppose $a_n = \frac{2n+1}{n+3}$. Find a_1 , a_2 , and a_3 .

2 SEQUENCE LIMIT

Evaluate $\lim_{n \rightarrow \infty} \frac{3n+2}{n+5}$.



Name: _____

Date: _____

Page 2

LEVEL 1 · SEQUENCES AND GEOMETRIC SERIES

3 GEOMETRIC SEQUENCE

Determine whether the sequence

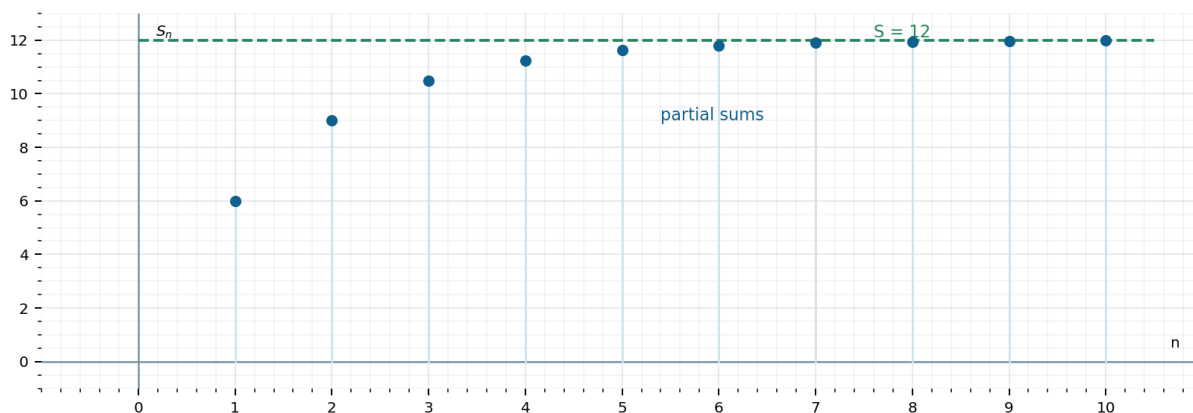
$$5, 10, 20, 40, \dots$$

is arithmetic or geometric. If geometric, find the common ratio. Choices: Arithmetic, $d = 5$ Geometric, $r = 2$
 Geometric, $r = 5$

4 INFINITE GEOMETRIC SERIES

Evaluate

$$6 + 3 + \frac{3}{2} + \frac{3}{4} + \dots$$



Name: _____

Date: _____

Page 3

LEVEL 2 · BASIC CONVERGENCE TESTS

5 NTH-TERM TEST

Determine whether $\sum_{n=1}^{\infty} \frac{3n}{2n+1}$ converges or diverges.

6 HARMONIC SERIES

Determine whether $\sum_{n=1}^{\infty} \frac{1}{n}$ converges or diverges. Choices: Converges to 1 Converges to 0 Diverges

7 p -SERIES

Determine whether $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ converges or diverges.

8 INTEGRAL TEST

Use the Integral Test to determine whether

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

converges or diverges.

Name: _____

Date: _____

Page 4

LEVEL 3 · COMPARISON AND ALTERNATING SERIES

9 DIRECT COMPARISON

Determine whether $\sum_{n=1}^{\infty} \frac{1}{n^2 + 4}$ converges or diverges.

10 LIMIT COMPARISON

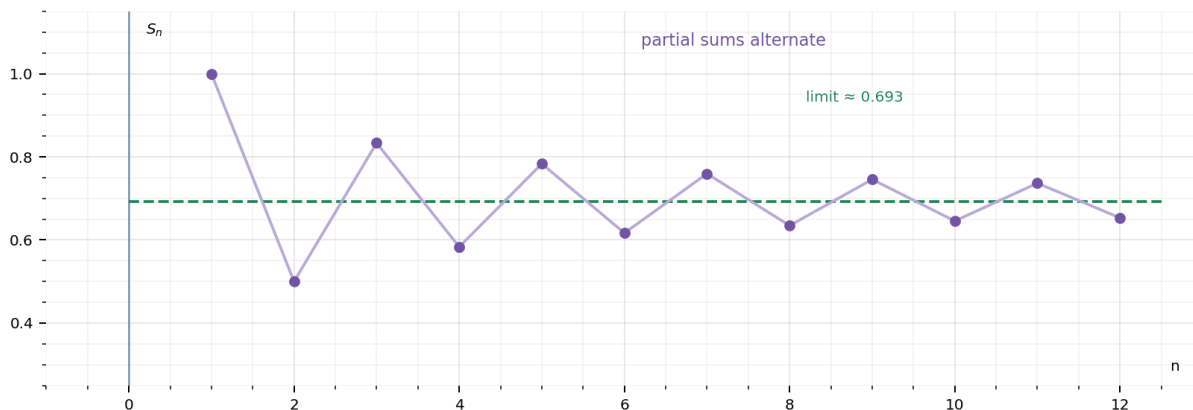
Use limit comparison with $\frac{1}{n}$ to determine whether

$$\sum_{n=1}^{\infty} \frac{4n+1}{n^2+3}$$

converges or diverges.

11 ALTERNATING SERIES

Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges or diverges.



12 CONDITIONAL CONVERGENCE

Classify $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ as absolutely convergent, conditionally convergent, or divergent. Choices: Absolutely convergent

Conditionally convergent Divergent

Name: _____

Date: _____

Page 5

LEVEL 4 · RATIO, ROOT, AND POWER SERIES

13 RATIO TEST

Determine whether $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ converges or diverges.

14 ROOT TEST

Determine whether

$$\sum_{n=1}^{\infty} \left(\frac{2n}{3n+1} \right)^n$$

converges or diverges.

15 INTERVAL OF CONVERGENCE

Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{x^n}{n}.$$



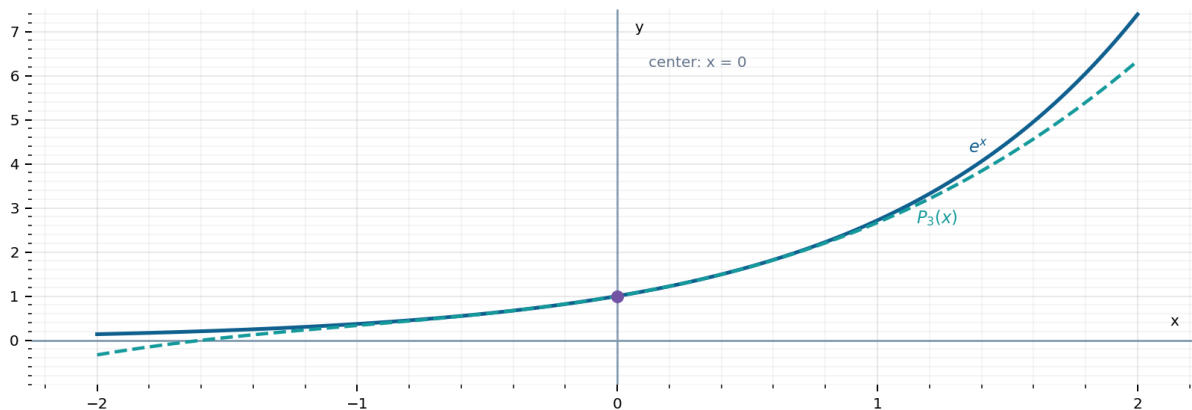
Name: _____

Date: _____

Page 6

LEVEL 5 · TAYLOR AND MACLAURIN SERIES

16 MACLAURIN POLYNOMIAL

Find the degree-3 Maclaurin polynomial for e^x .

17 COMMON SERIES

Which Maclaurin series represents $\cos x$? Choices: $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $1 + x + \frac{x^2}{2!} + \dots$

18 GEOMETRIC POWER SERIES

Write a power series for $\frac{1}{1+x}$. State its interval of convergence.

Name: _____

Date: _____

Page 7

Use the solutions to check your setup and reasoning.

ANSWER KEY · PROBLEMS 1-6

1 SEQUENCE TERMS

$$a_1 = \frac{3}{4}, \quad a_2 = 1, \quad a_3 = \frac{7}{6}. \quad \text{Answer: } \frac{3}{4}, 1, \frac{7}{6}$$

2 SEQUENCE LIMIT

$$\text{Divide by } n: \frac{3 + 2/n}{1 + 5/n} \rightarrow 3. \quad \text{Answer: } 3$$

3 GEOMETRIC SEQUENCE

Each term is twice the previous term. **Answer:** Geometric, $r = 2$

4 INFINITE GEOMETRIC SERIES

$$a = 6, r = \frac{1}{2}, \text{ so } S = \frac{6}{1 - 1/2} = 12. \quad \text{Answer: } 12$$

5 NTH-TERM TEST

$$\lim_{n \rightarrow \infty} \frac{3n}{2n+1} = \frac{3}{2} \neq 0. \quad \text{Answer: Diverges}$$

6 HARMONIC SERIES

This is the harmonic series, a p -series with $p = 1$. **Answer:** Diverges

Name: _____

Date: _____

Page 8

Use the solutions to check your setup and reasoning.

ANSWER KEY · PROBLEMS 7-12

7 p -SERIES $p = \frac{5}{2} > 1$. **Answer:** Converges

8 INTEGRAL TEST

 $\int_1^{\infty} \frac{dx}{x^2 + 1} = [\arctan x]_1^{\infty} = \frac{\pi}{4}$, finite. **Answer:** Converges

9 DIRECT COMPARISON

 $0 < \frac{1}{n^2 + 4} < \frac{1}{n^2}$ and $\sum 1/n^2$ converges. **Answer:** Converges

10 LIMIT COMPARISON

With $b_n = 1/n$, $\lim \frac{a_n}{b_n} = \lim \frac{n(4n+1)}{n^2+3} = 4$. The harmonic series diverges. **Answer:** Diverges

11 ALTERNATING SERIES

 $b_n = 1/n$ decreases and $b_n \rightarrow 0$. **Answer:** Converges by the Alternating Series Test

12 CONDITIONAL CONVERGENCE

The alternating harmonic series converges, while $\sum |a_n| = \sum 1/n$ diverges. **Answer:** Conditionally convergent

Name: _____

Date: _____

Page 9

Use the solutions to check your setup and reasoning.

ANSWER KEY · PROBLEMS 13-18

13 RATIO TEST

$$L = \lim_{n \rightarrow \infty} \frac{3}{n+1} = 0 < 1. \quad \text{Answer: Converges absolutely}$$

14 ROOT TEST

$$L = \lim_{n \rightarrow \infty} \frac{2n}{3n+1} = \frac{2}{3} < 1. \quad \text{Answer: Converges absolutely}$$

15 INTERVAL OF CONVERGENCE

Ratio test gives $|x| < 1$. At $x = 1$, $\sum 1/n$ diverges. At $x = -1$, $\sum (-1)^n/n$ converges. **Answer:** $[-1, 1)$

16 MACLAURIN POLYNOMIAL

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots. \quad \text{Answer: } P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

17 COMMON SERIES

Cosine uses even powers with alternating signs. **Answer:** $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

18 GEOMETRIC POWER SERIES

Use $\frac{1}{1-r} = \sum r^n$ with $r = -x$: $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$, valid for $|x| < 1$. **Answer:** $-1 < x < 1$